原子

・ドブロイ波長：$\lambda = \dfrac{h}{mv}$

・エネルギー$h\nu$

・運動量$p = \dfrac{h\nu}{c} = \dfrac{h}{\lambda}$

・コンプトン効果

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$$

エネルギー保存則

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2}mv^2$$

運動量保存則

$$\frac{h}{\lambda} = \frac{h}{\lambda'} + mv \Rightarrow \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 = m^2v^2 \Rightarrow \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'} = m^2v^2$$

$$\begin{cases} \dfrac{h}{\lambda} = \dfrac{h}{\lambda'}\cos\phi + mv\cos\theta \\ 0 = \dfrac{h}{\lambda'}\sin\phi - mv\sin\theta \end{cases}$$

$$\cos\theta = \frac{1}{mv}\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\phi\right)$$

$$\sin\theta = \frac{1}{mv}\frac{h}{\lambda'}\sin\phi$$

$$(mv)^2(\cos^2\theta + \sin^2\theta) = \left(\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'}\cos\phi\right)$$

$$(mv)^2 = 2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \left(\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'}\cos\phi\right)$$

$$\lambda' - \lambda = \frac{h}{2mc}\left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} - 2\cos\phi\right)$$

$$\lambda \cong \lambda' \Rightarrow \frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} \cong 2$$

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$$

$$hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=\frac{1}{2}mv^2$$

$$hc\frac{\lambda'-\lambda}{\lambda\lambda'}=\frac{1}{2}mv^2$$

$$\frac{hc}{\lambda\lambda' mv^2}=\frac{1}{2(\lambda'-\lambda)}$$

$$\frac{hc}{\lambda\lambda' mv^2}(1-\cos\phi)=\frac{1}{2(\lambda'-\lambda)}(1-\cos\phi)$$